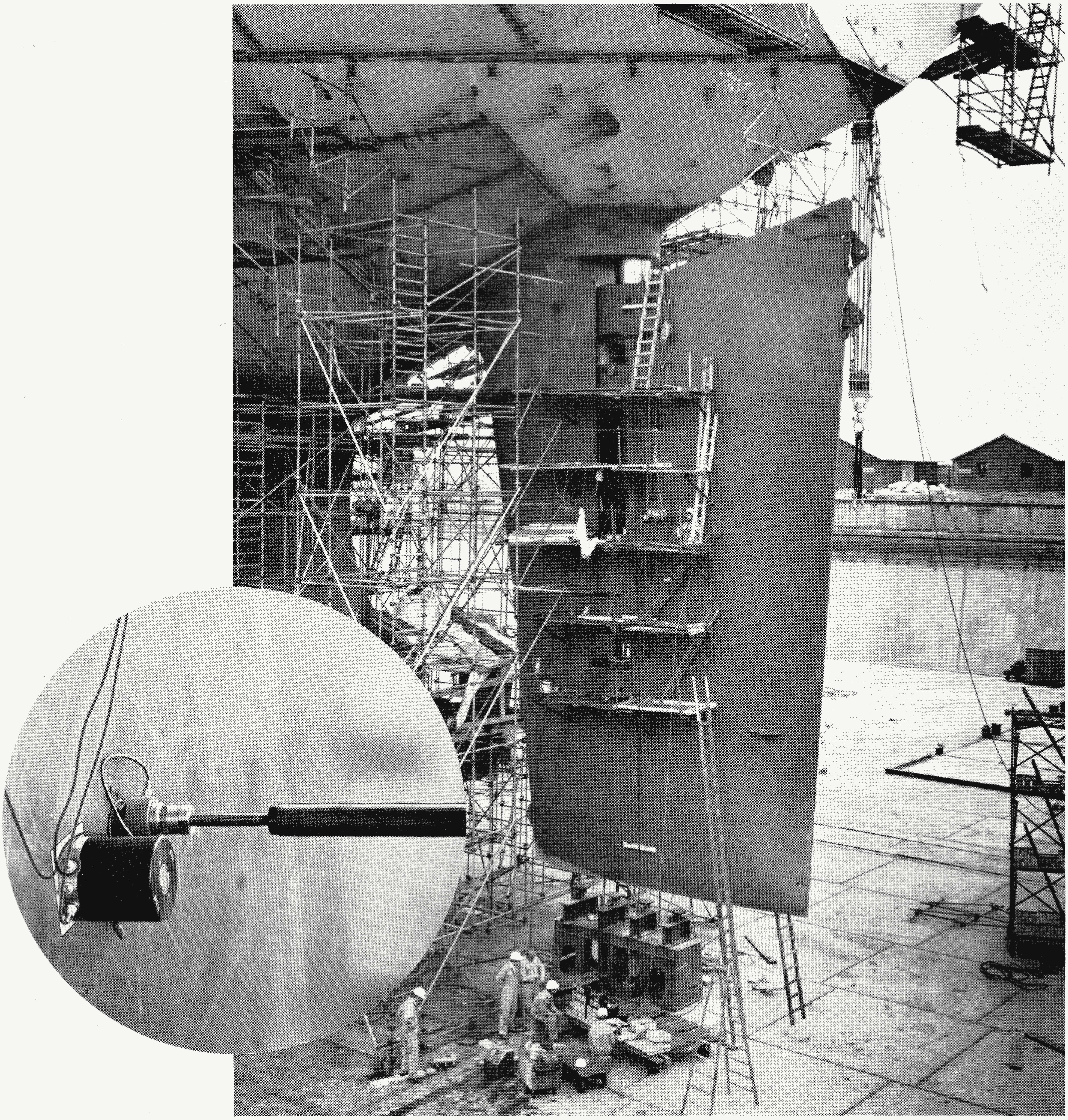
BRÜEL & KJÆR
NÆRUM · DENMARK

BRÜEL & KJÆR application notes

Measurement of the Dynamic Properties of Materials and Structures.



Measurement of the Dynamic Properties of Materials and Structures

By Hans P. Olesen

Forced vibration techniques provide very powerful tools for the investigation of structures, structural elements, and the materials from which the structural elements were formed.

Some very important concepts in this type of investigation are those of mechanical impedance and mechanical mobility. They were developed from the first electro-mechanical and electroacoustical analogies which were applied in the 1920's. In the 1930's and 1940's they were used very much for the development and refinement of vibration isolation in submarines and airplanes.

In the years since then the use of the impedance concept has increased steadily both because the understanding of the theory has been improved and because the possibilities for measurement of the values have been developing rapidly, especially in recent years.

One of the main advantages of using the impedance or mobility theory is to avoid the solution of a classical set of differential equations. This is often achieved by application of electrical circuit theory to an electrical analogy of the system in question.

However, although electrical analogies are very useful for mixed electro-mechanical systems it is also convenient to have rules developed for and applicable to purely mechanical systems. This has been consistently pursued in the development of vibration theory in "Steady State Vibration" by Salter. (1).

Mr. Salter shows how mechanical systems can be simplified in a way similar to electrotechnical theory to yield fast reliable analysis by means of the combination of the simple responses of the three fundamental elements, mass, spring and damper.

Definitions

The mechanical impedance and mobility for simple harmonic motion are defined as the complex ratios of force to velocity and velocity to force respectively. This is shown in Table 1 where in addition, the similar ratios involving acceleration or displacement are given.

Dynamic Mass $= \frac{F}{a}$	(acceleration through force) $= \frac{a}{F}$
(Apparent Weight)	
Mechanical Impedance $= \frac{F}{v}$	Mobility $= \frac{v}{F}$
	(Mechanical Admittance)
Stiffness $= \frac{F}{d}$	Compliance $= \frac{d}{F}$

073039

Table 1. Terminology for complex dynamic ratios of force and motion

The terms given in the table are taken from the American Standard USAS S2.6-1963: Specifying the Mechanical Impedance of Structures (2). Other terms have been used in some cases, for some of the values given in the table but they are not given here.

When force and motion values are measured at the same point and in the same direction the ratios are termed driving point values or short point values, i.e., point impedance.

Practical measurements

When force and motion are measured at different points or at the same point with an angle between them they are termed transfer values, i.e., transfer impedance.

In practical measurements a choice must be taken of which ratio should be measured. In many cases the choice is not critical, and any desired parameter may be chosen.

However, each of the ratios in Table 1 has its own advantages and it may be selected for certain applications.

The easy comparison with mathematical models will often give preference to either mechanical impedance or mobility. Similarly, the dynamic mass, and acceleration divided by force both take full advantage of the accelerometer dynamic range. For small systems the ratios in the left half of the Table may be chosen to ensure constant motion and for very large systems one of the ratios of the right hand side of the Table are to be chosen because of their demand for constant force. Thereby maximum signal level can be provided over the whole frequency range for a given vibration exciter.

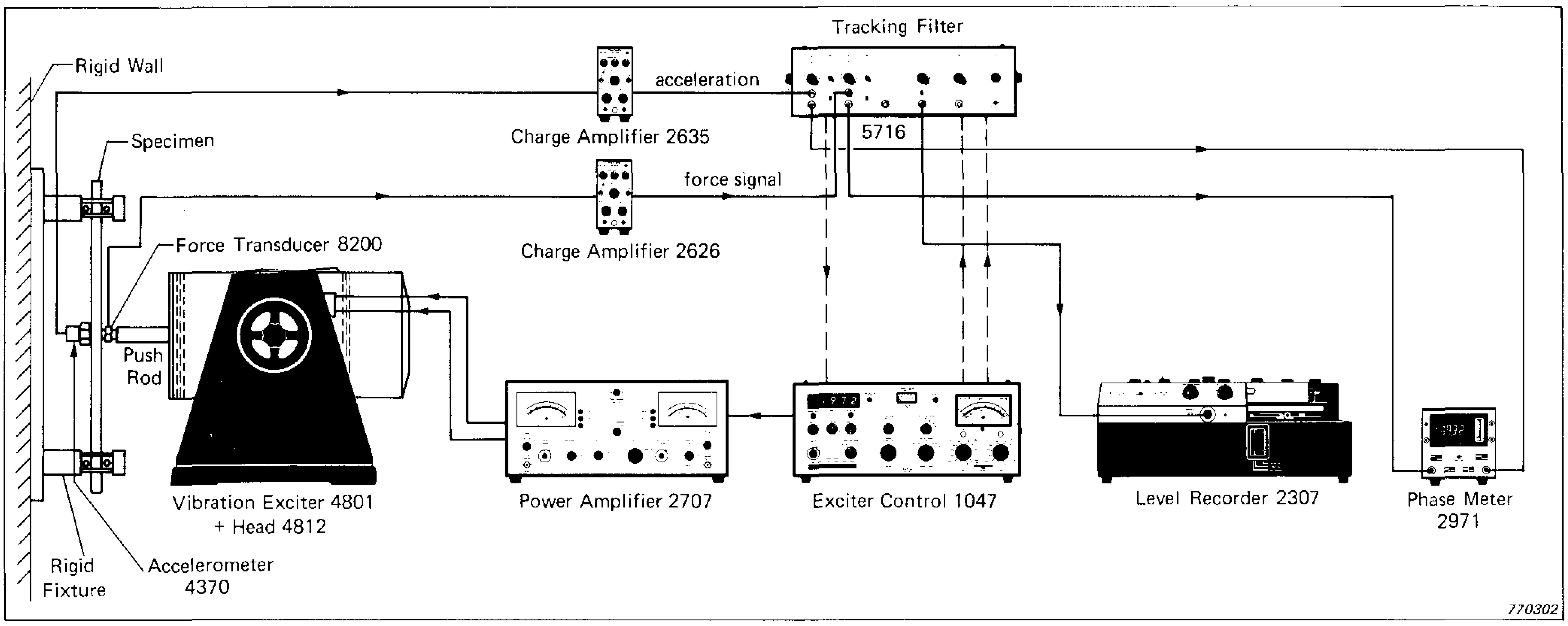


Fig.1. Measurement arrangement suitable for measurement of the ratios given in Table 1

Measurement of Modulus of Elasticity

In Fig.1 is shown a measurement arrangement used to measure the stiffness of asphalt bars produced as test specimens, or cut from a finished road.

In this application which is fully described in Ref.(4) the stiffness was chosen to be recorded, as a constant displacement was considered to come nearest to the conditions for asphalt in a finished road. The measurements were carried out to determine the modulus of elasticity of asphalt at frequencies below the first bending resonance and, therefore, both the stiffness and the phase between force and motion were needed.

The measurement set-up represents in fact a typical system for measuring mechanical impedance, mobility or other ratios. It contains an Exciter Control which delivers a sinusoidal signal to a Power Amplifier and thereby to the Vibration Exciter which directs a vibrational force through a Force Transducer to the application point. The motion is picked up by an Accelerometer which can be placed at the force application point to measure the point value (as used in the above case), or at another point to measure the transfer value. The acceleration and force signals are amplified in Conditioning Amplifiers and filtered in a two channel constant bandwidth Tracking Filter which delivers filtered signals to a Phase Meter and to the Exciter Control. The Exciter Control contains control circuits and integrating circuits to provide constant force, acceleration, velocity or displacement at the control point and to deliver any one of these signals to a graphic Level Recorder.

This system may, naturally, be reduced for simpler applications or some of the instrumentation may be interchanged with other instruments which may provide optimal solutions to other measurement problems, (see also Ref.(3)).

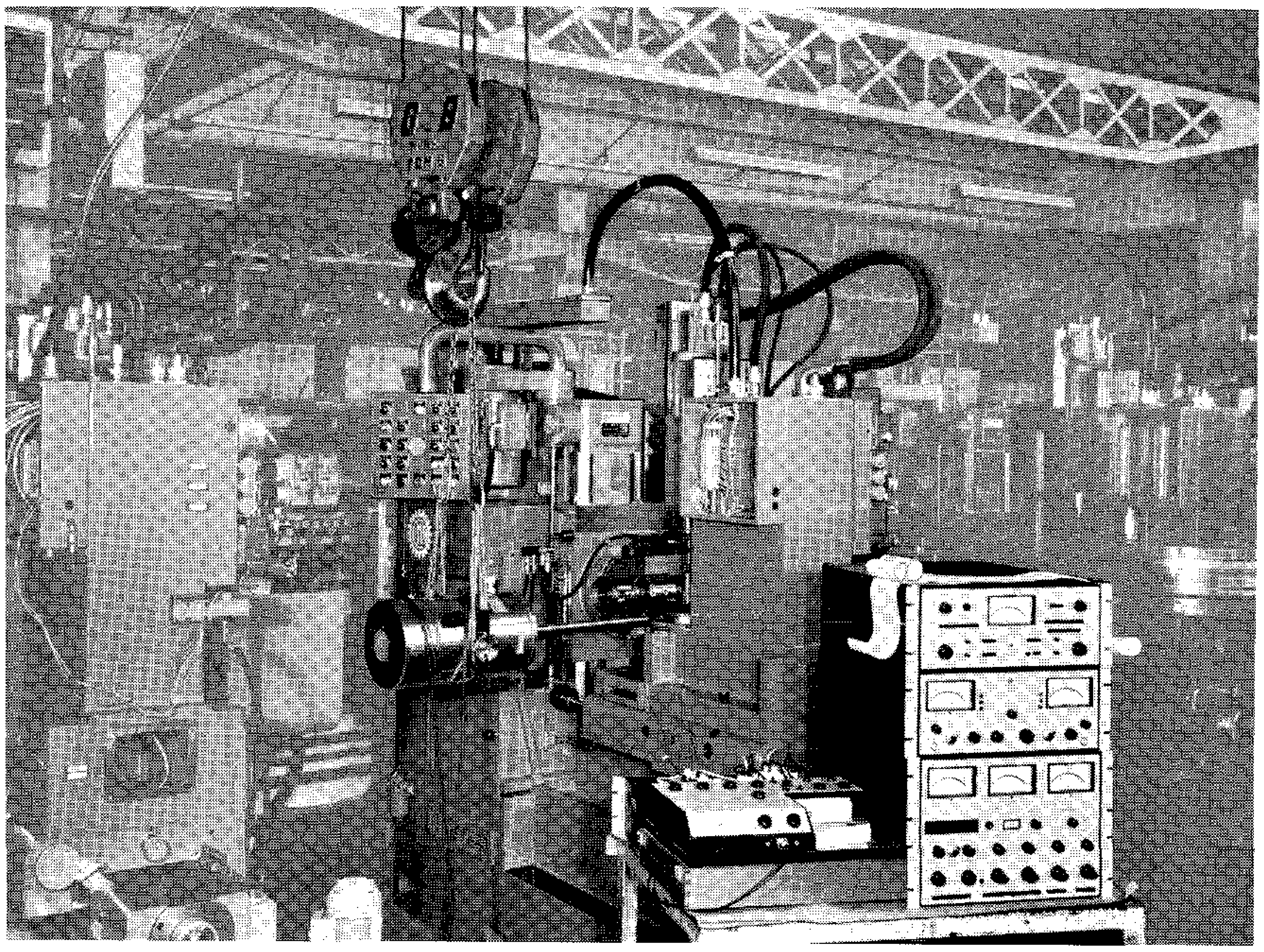


Fig.2. Photograph of a CIMA hobbing machine subjected to a constant force from a vibration exciter

In applications where measurements are to be taken on very light structures, an electronic mass compensation unit must be introduced to compensate for that part of the Force Transducer mass which is between the active elements and the structure. In such cases the Force Transducer may advantageously be interchanged with an Impedance Head which combines both force transducer and accelerometer in the same housing. This unit, naturally, must also be used in conjunction with a suitable mass compensation when its 1 gramme mass below the active elements may introduce errors in the measurements.

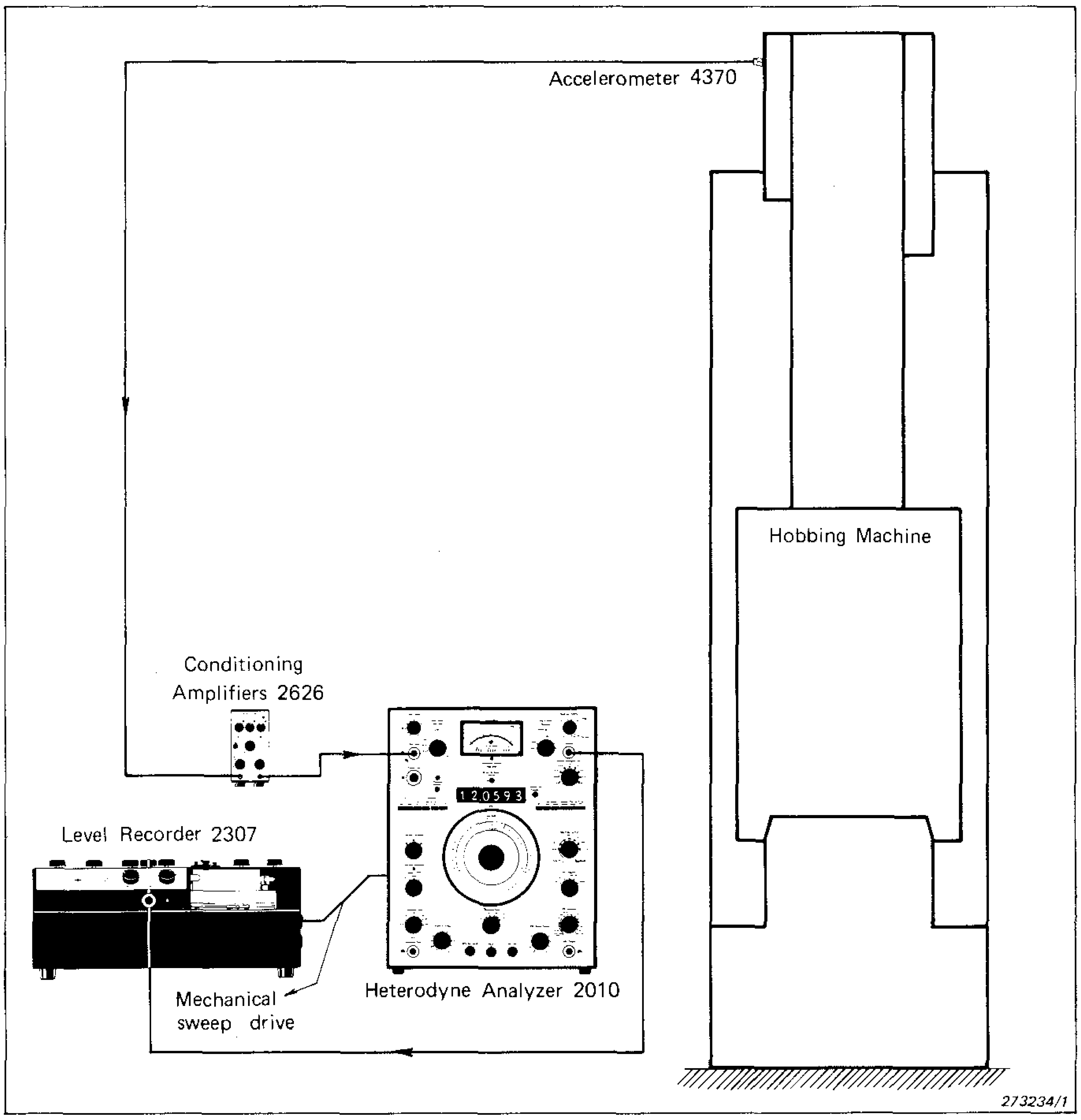


Fig.3. Measurement arrangement for frequency analysis of the signal from the hobbing machine during operation

The properties of a hobbing machine

Fig.3 shows the measurement arrangement used for a narrow band frequency analysis carried out during the cutting of a gear wheel. The vibration at several points was analyzed to obtain information about transverse and longitudinal vibration levels at different frequencies. The results measured at one point are given in Fig.4.

As mentioned above the limiting factor in measurement on heavy structures is the available force.

This was partly the case for measurements on a CIMA hobbing machine which were carried out at Bologna in 1971. Here measurements were taken with the instrumentation and arrangements shown in Figs.2, 3 and 5.

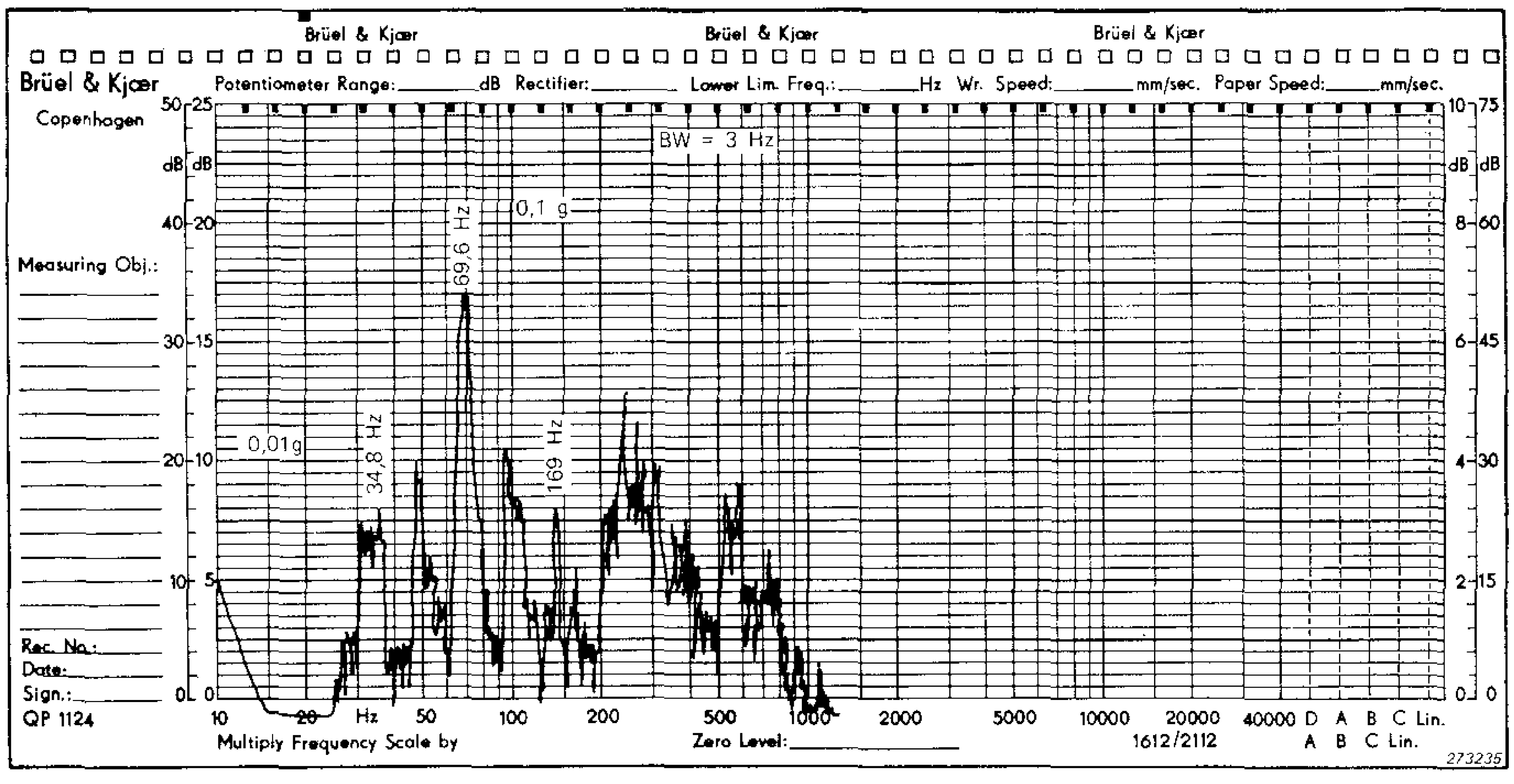


Fig.4. Recorded frequency analysis from the hobbing machine

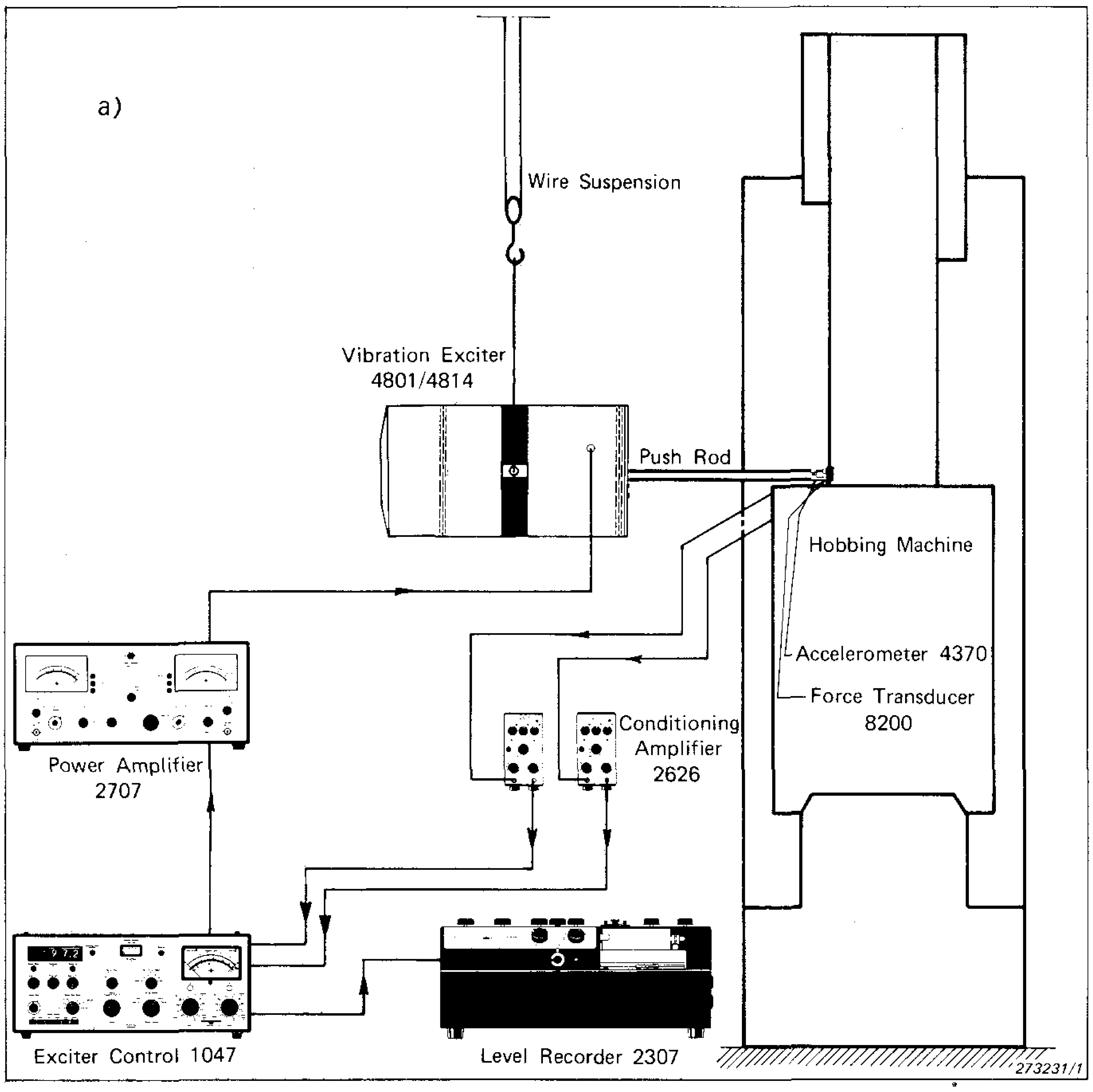


Fig.5a. **Measurement arrangement for measurement of acceleration as a function of frequency at constant force for the hobbing machine**

To obtain knowledge about the structural response of the hobbing machine and to compare this with the results from frequency analyses the machine was excited by a Vibration Exciter by means of a push rod while the exciter was suspended from a crane. The arrangement is shown in Fig.5.

The instrumentation is similar to that of Fig.1 except that a smaller Vibration Exciter is used and that the Phase Meter and the Tracking Filter is omitted. The force was applied first to a gear wheel at 90° and 45° angle to the machine axis as indicated in Fig.5b. The mass of the vibration exciter acted as reaction to the applied force which was kept constant over the frequency range from 10 to 1000 Hz.

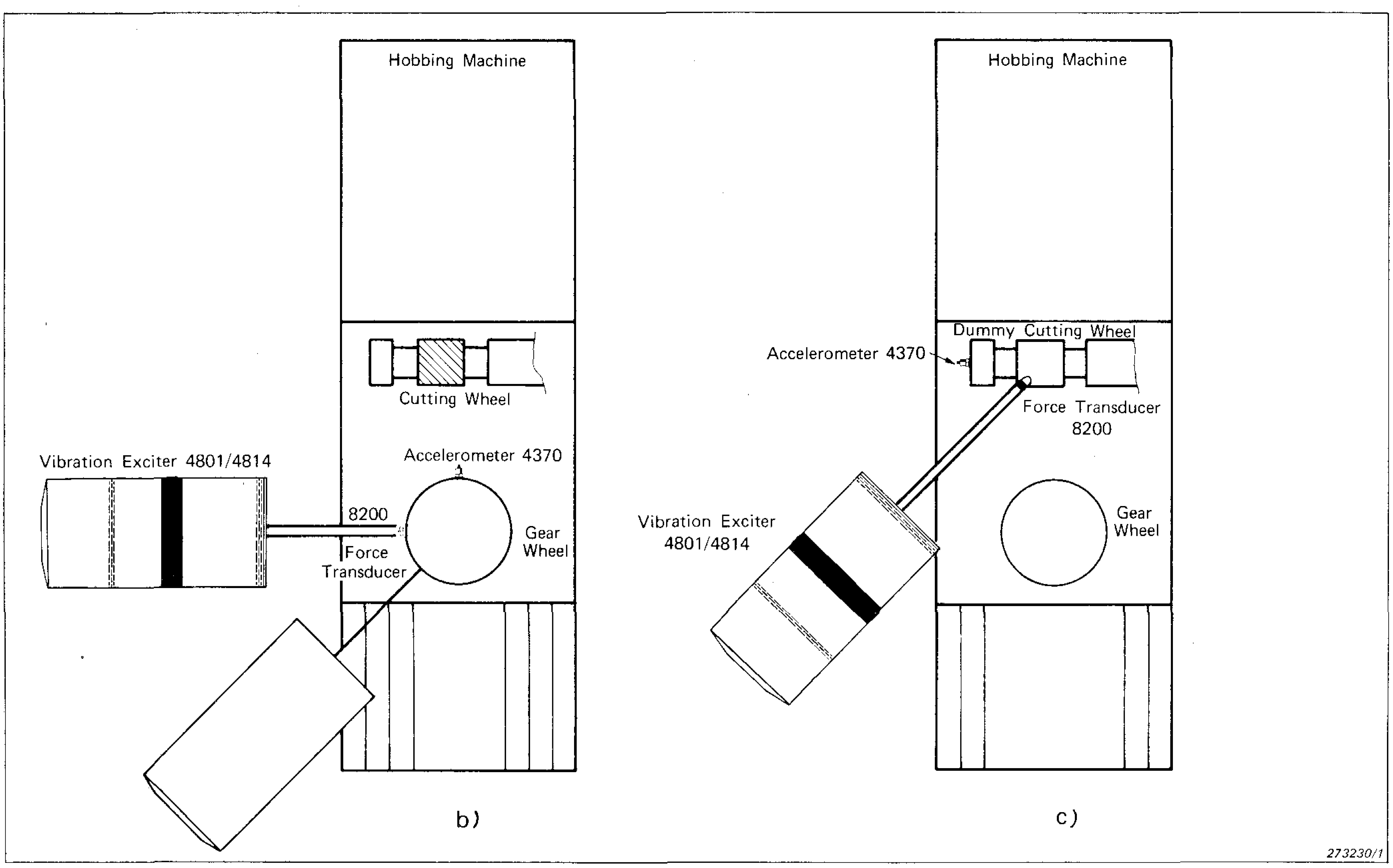


Fig.5b and c. The measurement arrangement of Fig.5a. as seen from above

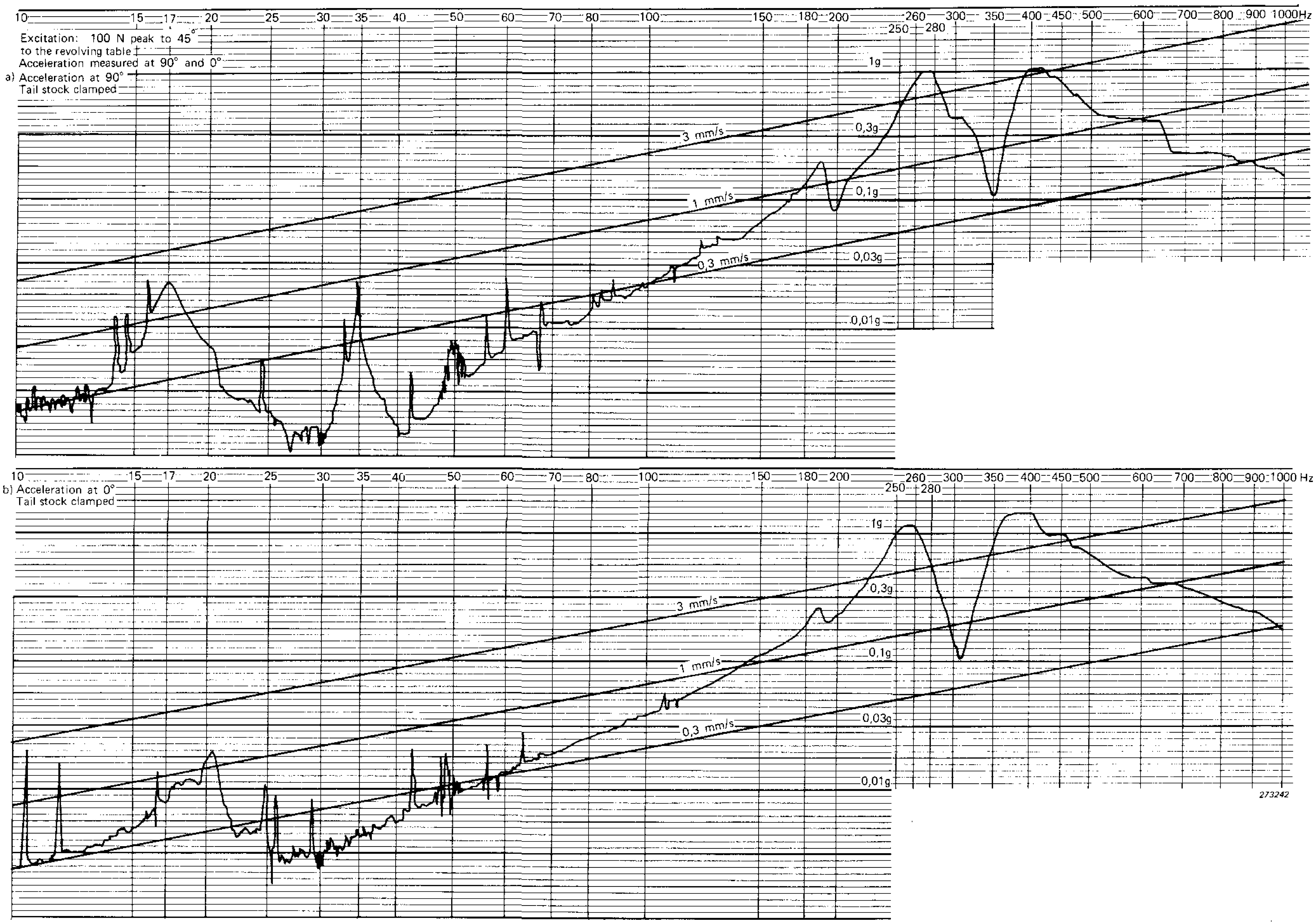


Fig.6. Acceleration as a function of frequency for a constant force applied at 45° to the machine axis. Above: In the transverse direction, below: In the longitudinal direction

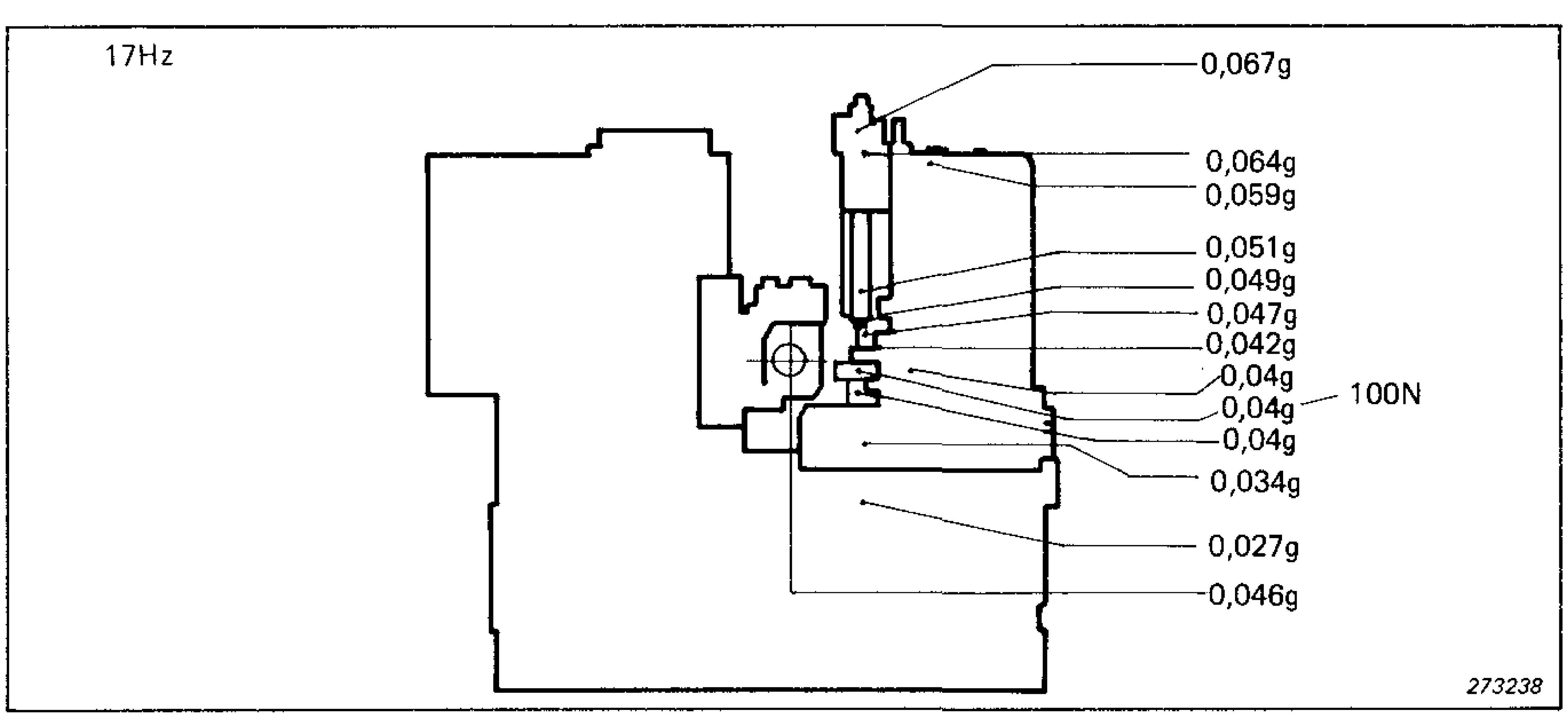


Fig.7. The acceleration levels at 17 Hz measured in transverse direction at different points of the machine

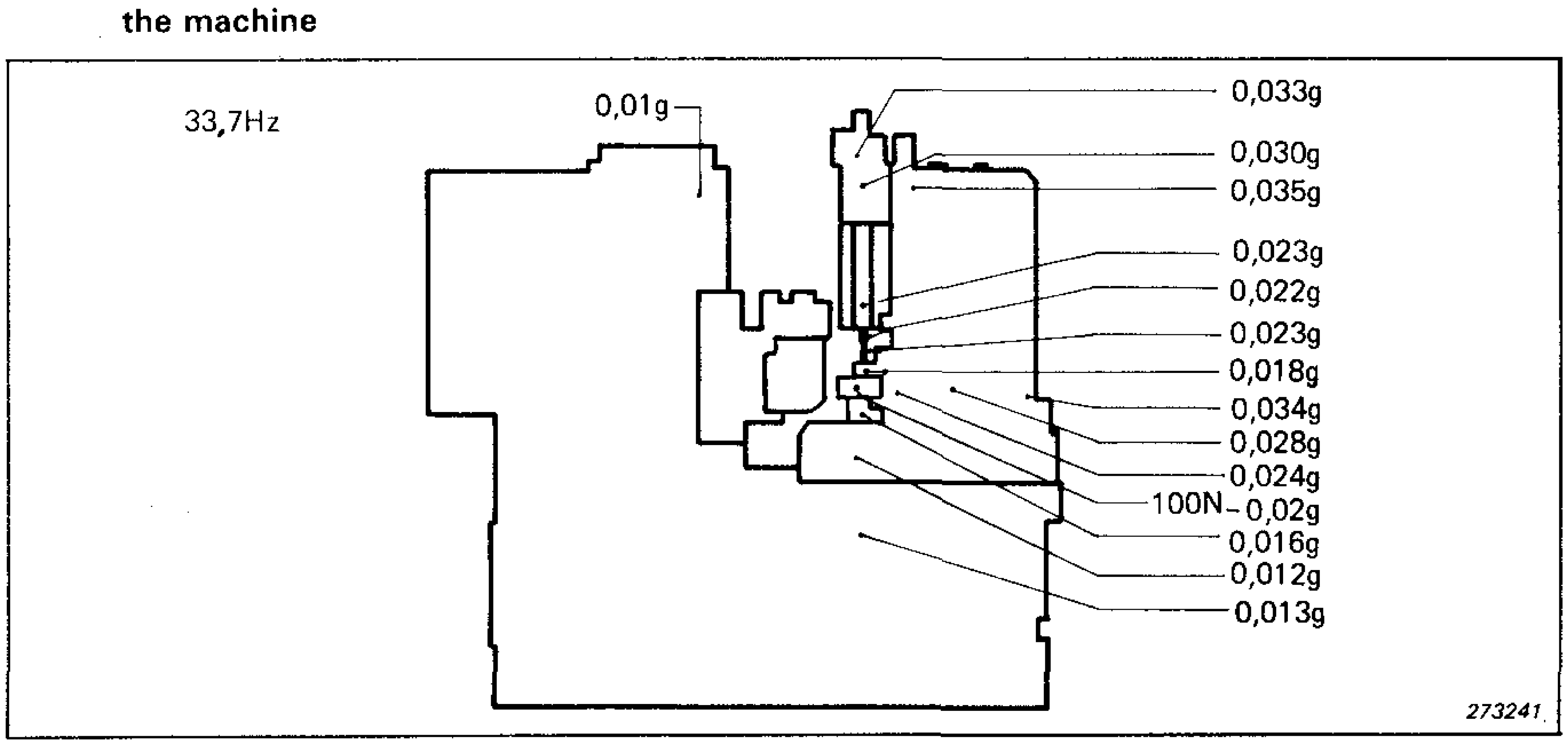


Fig.8. The acceleration levels at 33,7 Hz in transverse direction

For each force application mode the application point accelerations at 90° and 0° for constant force of 100 N were recorded as shown in Fig.6 for 45° excitation of the gear wheel. It is seen that a number of frequencies show resonance peaks.

The frequency sweeps of the vibration exciter were stopped at some of these frequencies while acceleration was measured at several points across the hobbing machine;

The results of transverse vibration at 17 Hz are shown in Fig.7. They indicate that 17 Hz is a rocking frequency of the whole machine which is not interesting for this investigation.

Fig.8 shows the figures for 33,7 Hz. It is seen that the resonance is related to the machine tailstock.

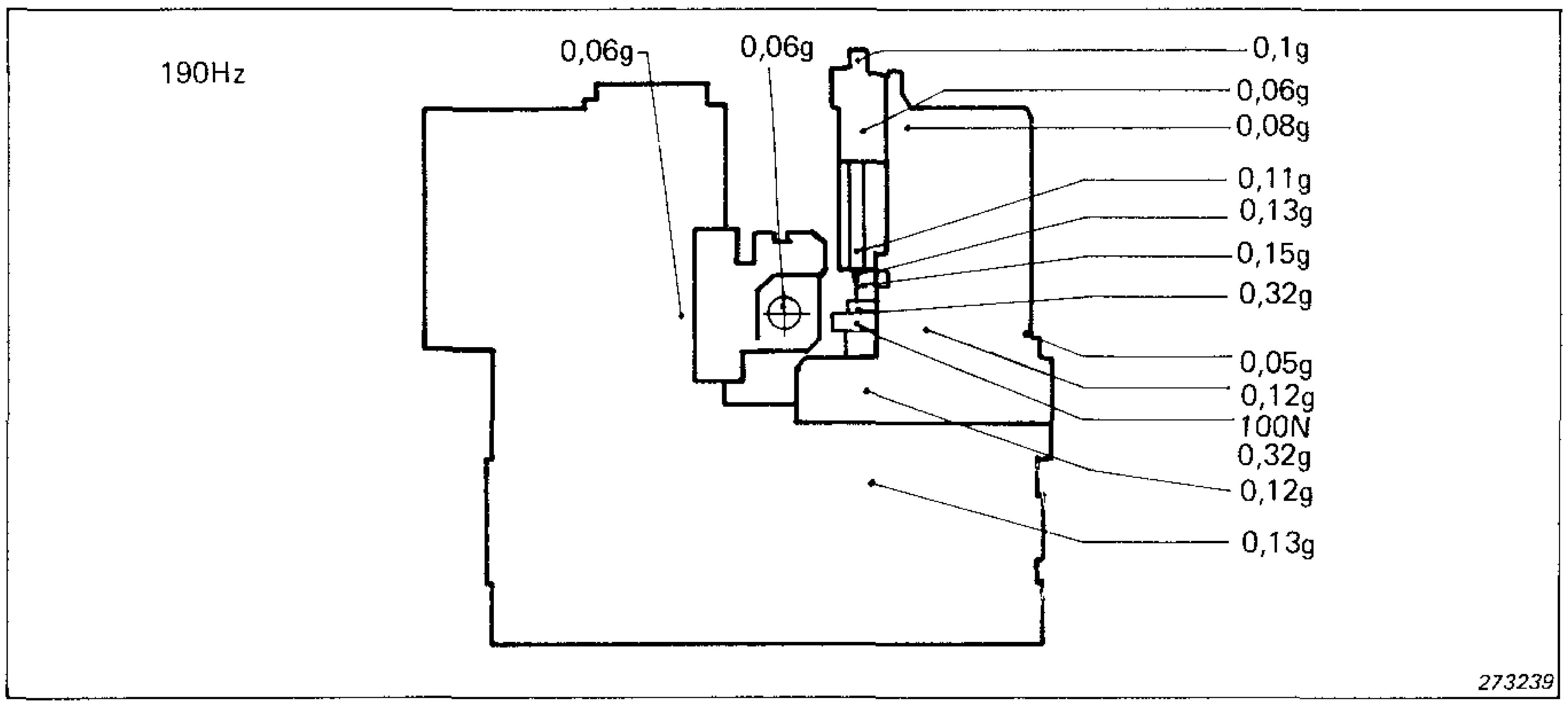


Fig.9. The acceleration levels at 190 Hz in transverse direction

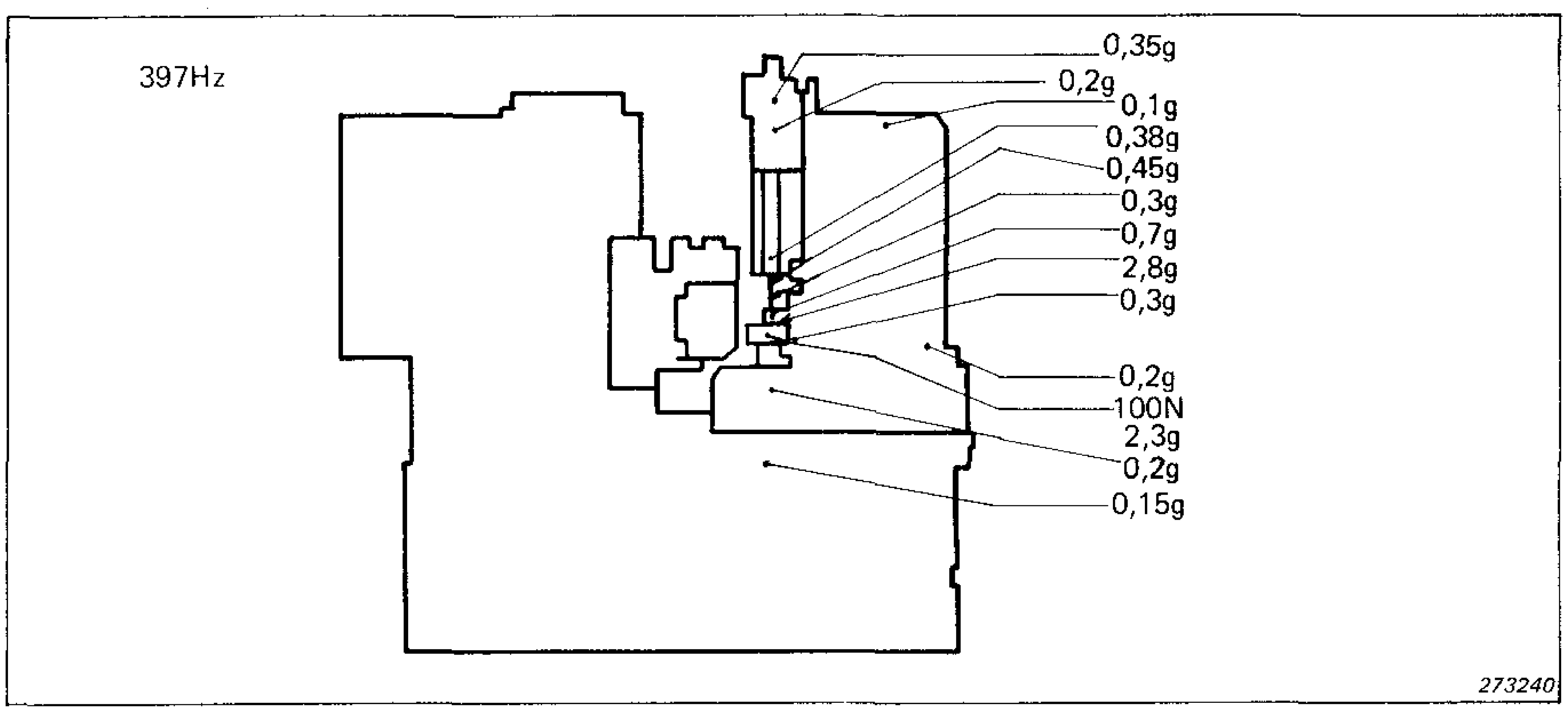


Fig.10. The acceleration levels at 397 Hz in transverse direction

Figs.9 and 10 show the results for 190 and 397 Hz. Both show indication of resonance in the gear wheel fixture.

Similar measurements were conducted at the cutting wheel dummy, for excitation at 45° to the machine's axis. The acceleration was measured at 0° and 90° to the machine's axis and two such recordings are shown in Fig.11 for excitation of the cutting wheel dummy.

From the lowest curve it is seen that a rocking frequency is present in the longitudinal axis at 20,9 Hz. At 82 Hz there is a strong longitudinal vibration caused by a resonance of the hobbing wheel support. None of the resonances seem to be hit by any frequency in the measured frequency spectrum (Fig.4) and only further analysis, including phase *measurements would reveal if the* construction could be further optimized.

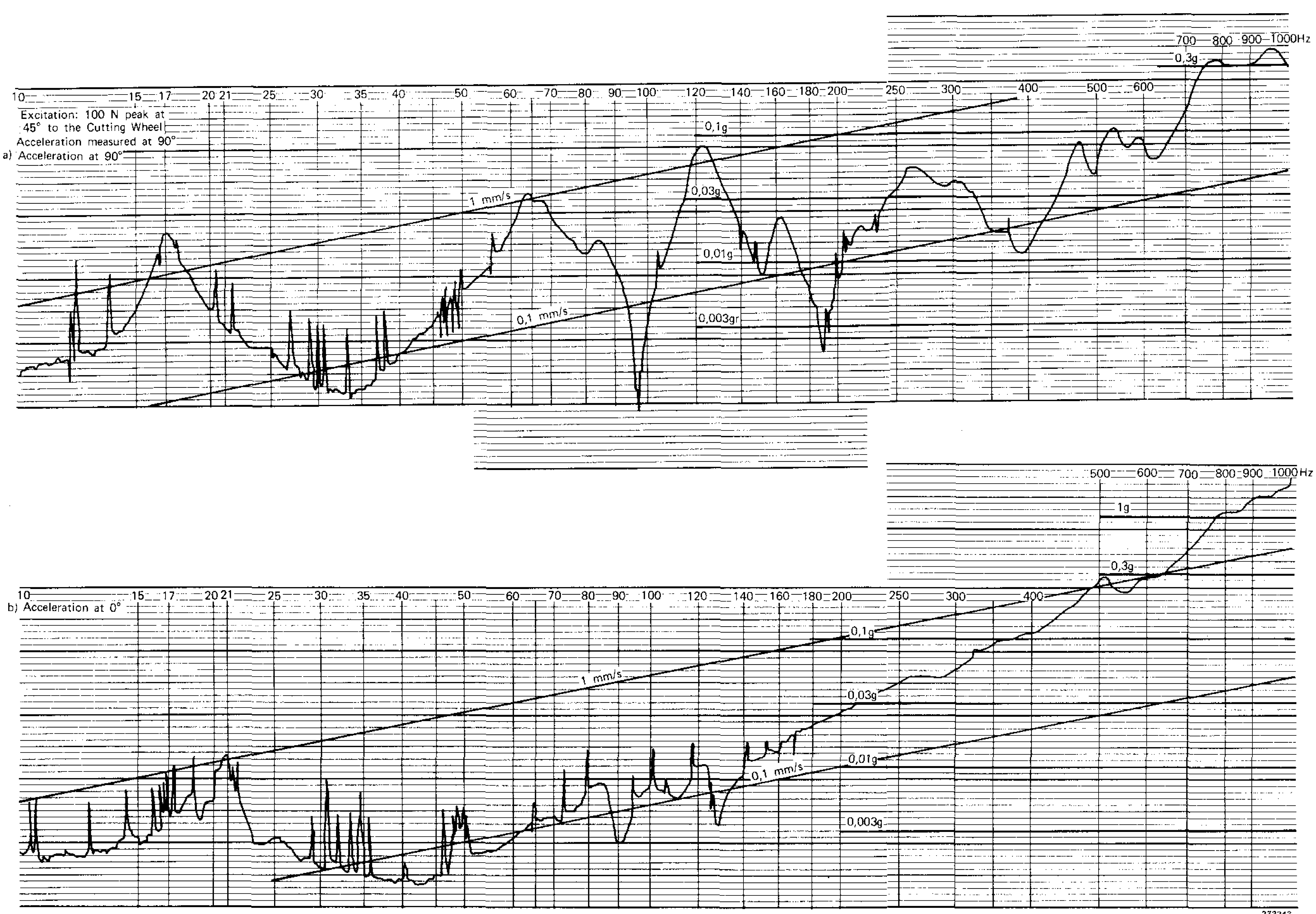


Fig.11. Acceleration as a function at frequency for a constant force applied to the dummy cutting wheel at 45° to the machine axis. Above: Transverse direction, and below: Longitudinal direction

Rudder resonances of a large tanker

Measurements were carried out in June 1973 at Odense Staalskibsværft A/S, Lindø to verify calculated resonances of the rudder of a 285000 t tanker. The measurement arrangement is given in Fig.12, and by the cover photograph.

A Vibration Exciter is suspended by wire straps from the scaffolding on the top of the rudder. The exciter mass acts as reaction to a constant force of 1000 N which is directed into the side of the rudder via a push rod and a Force Transducer as seen on Fig.13. The resulting acceleration signals at different points on the rudder are picked up by Accelerometers Type 4370 and 8306.

Both force and acceleration signals are led via Conditioning Amplifiers Type 2626 to a two channel Tracking Filter from which the force signal is led to the measuring circuit and the compressor of the Exciter Control Type 1047 which feeds the Vibration Exciter via a Power Amplifier. From the other Filter channel the acceleration signal is led to the Level Recorder Type 2307 on which the signal level is recorded in a decibel scale. Taking two signals from the Tracking Filter to a Phase Meter allows phase measurement between force and acceleration (or between two acceleration signals if the force signal bypasses the Tracking Filter). The phase measurements can be recorded on the Level Recorder in a separate run.

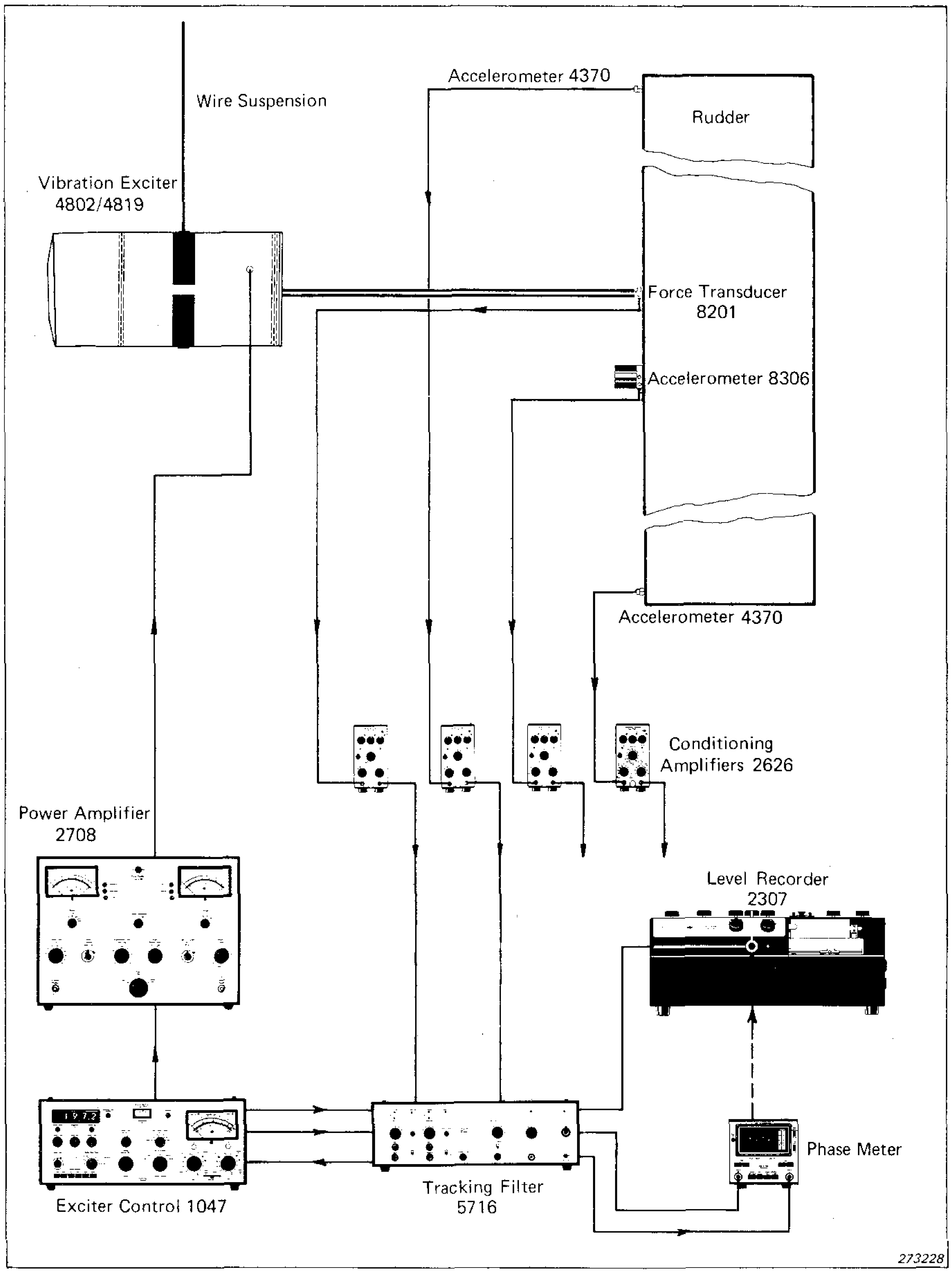


Fig.12. Measurement arrangement used to obtain the point and transfer mobilities of a ships rudder

For the purpose in question the signal frequency was varied with a sweep speed of 0,1 Hz/s in the frequency range from 4 Hz to 30 Hz while recording the accelerations for constant force from the rudder tip, the rudder bottom and the rudder horn, as well as from the force application point (see Fig.14).

The recorded accelerations (examples in Fig.15), were used to construct a diagram (Fig.16) from which the point and transfer mobilities measured at the different points could be compared.

From this diagram it is seen that at 7,5 Hz the force application point experiences an antiresonance while the transfer mobilities showed peaks at this frequency, thereby indicating that the rudder is vibrating around an axis through this point. This can be confirmed by using the phase measurements (example in Fig.17) together with the acceleration values to draw figures of the acceleration modes at certain frequencies. Such figures are shown for four frequencies in Fig.18 and it is clearly seen that the vibration modes are changing rapidly as the frequency is varied across the antiresonance at 7,5 Hz.

A further knowledge can be obtained of the stiffnesses and masses, as well as of rotary inertias involved in the rudder and rudder post construction from a comparison of the measured values with calculated rudder models.

Further measurements of rudder response will be carried out by the shipyard laboratory during the trial runs. These tests on a rudder in normal submerged condition with propeller excitation will, in conjunction with the above mentioned results, reveal information about the water mass to be accounted for in the calculation of future new constructions.

An improved knowledge of these factors has a great economic importance. Even very slight decreases in service speed because of rudder resonances that coincide with propeller frequencies, would reduce significantly, the profit of the ship operators, and thereby the competitive situation for the shipyard would become less favorable.

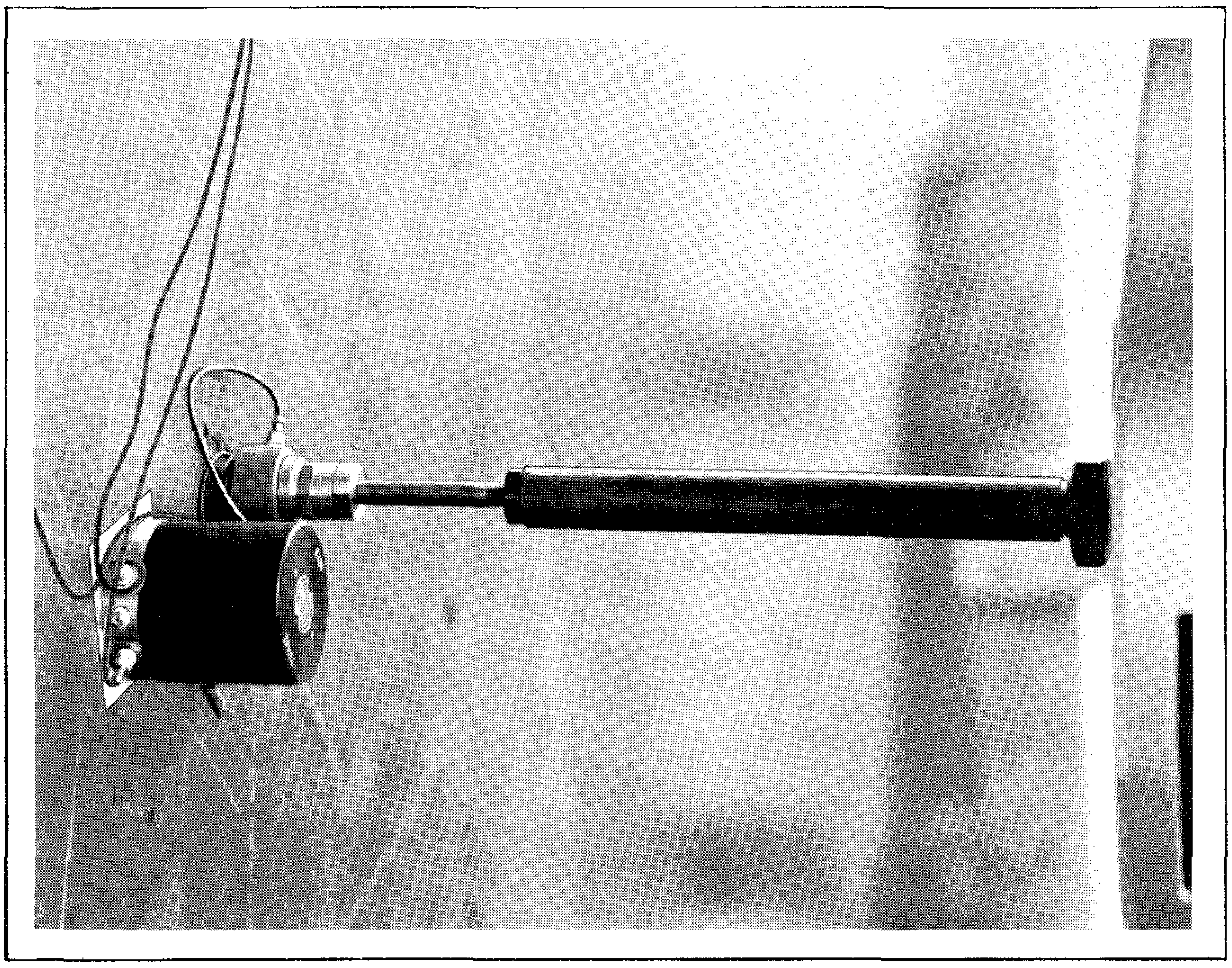


Fig.13. Close-up photograph of the arrangement of push rod and force transducer as well as the accelerometer Type 8306

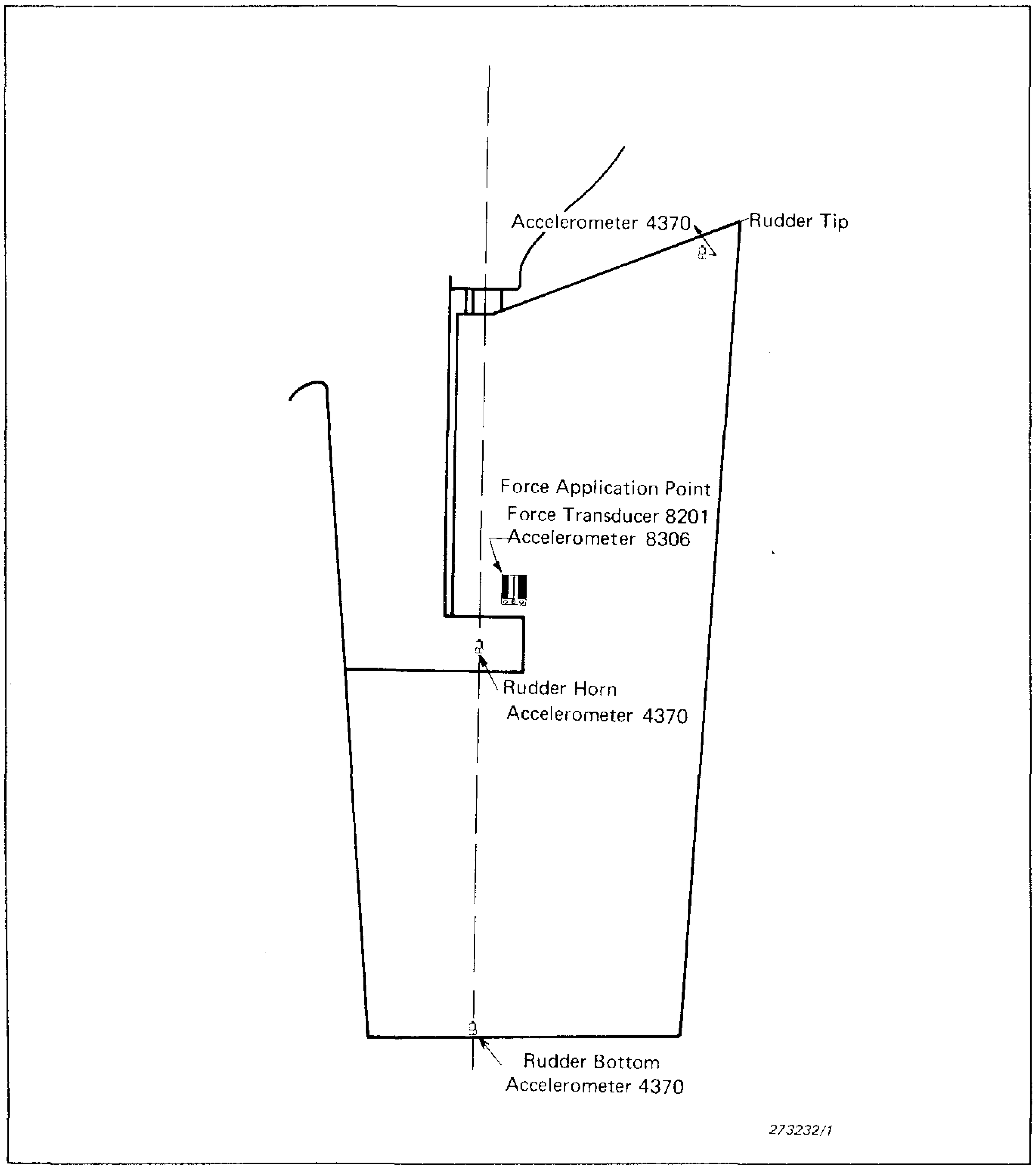


Fig.14. The placement of the accelerometers on the rudder side

Measurements of the forces transmitted into a foundation

Similar measurement technique was utilised in measurements on large pumps and turbines in a hydropower plant where the longitudinal and transverse vibration velocities of the bearing supports were frequency analyzed. The results were compared with mobility measurements at the same points to calculate, at each significant frequency, the force transmitted into the concrete foundation. Thereby a better estimate of the necessary dynamic strength of the foundation can be obtained and considerable savings may be achieved as the building costs represent a large percentage of total costs.

Discussion

The above examples have been concentrated mostly on the measurement on large structures. The results indicate that even much larger structures or buildings may be investigated with good results by means of the same instrumentation.

However, for measurements on very large structures there is a definite requirement for either low background vibration (and noise) or, in some cases, for greater vibration exciter force.

In the above cases the suspension of the Vibration Exciter in wire straps has proved very successful although care must be taken to ensure that string vibrations don't disturb the measurements. For application at very low frequencies extra mass may have to be added to the shaker body in order to ensure full force capability. Consider, for example the Exciter Body Type 4802 combined with the Mode Study Head Type 4819 which was used to excite the ships rudder. The combination has a mass of app. 200 kg, and the Mode Study head has a maximum peak to peak displacement of 38,1 mm or 19 mm peak, and a full force capability of 1450 N peak. The small force needed to deflect the built-in flexures is disregarded, and the structure to be vibrated is considered to have much smaller motion that the Vibration Exciter. 1450 N excite

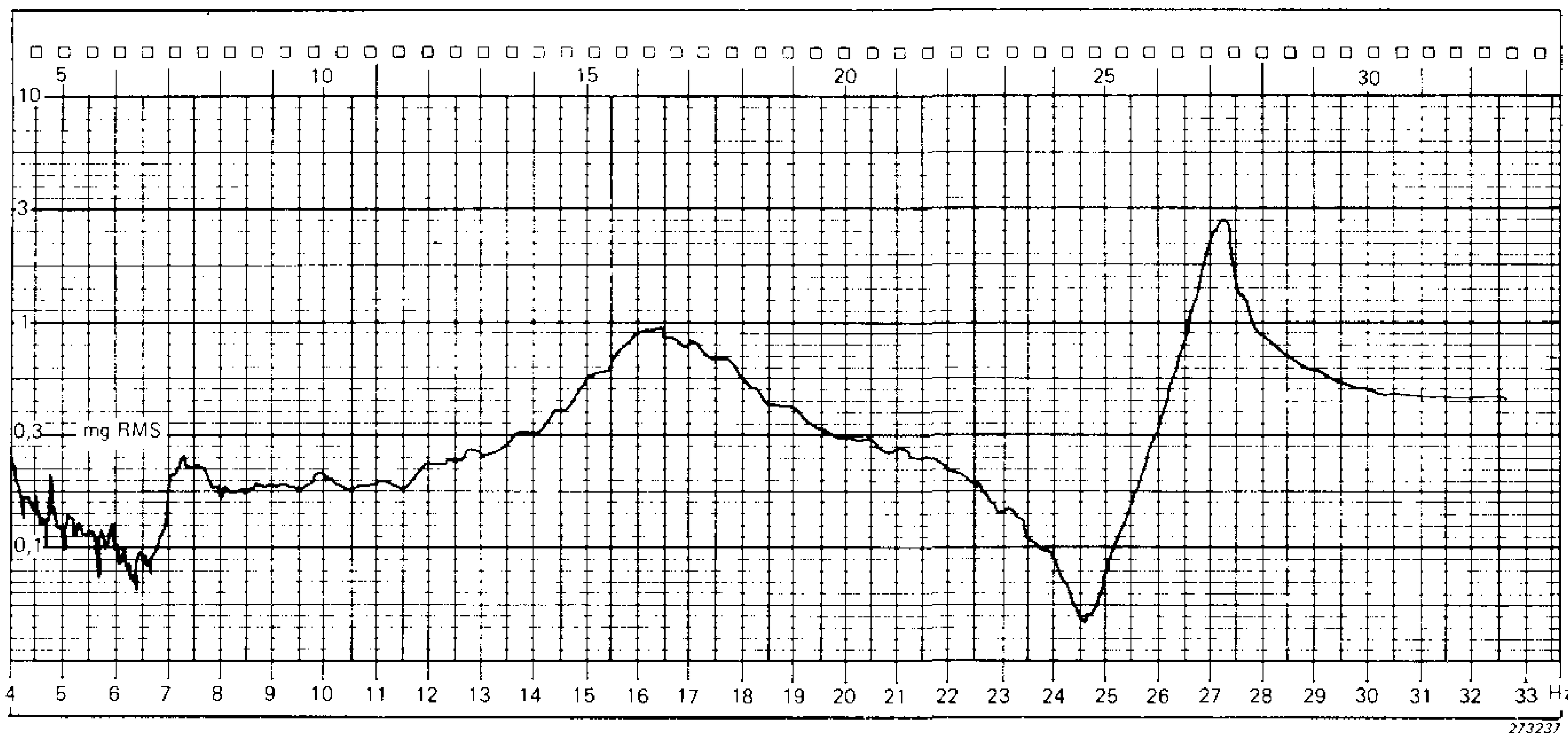


Fig.15. The acceleration recording as obtained from the Level Recorder

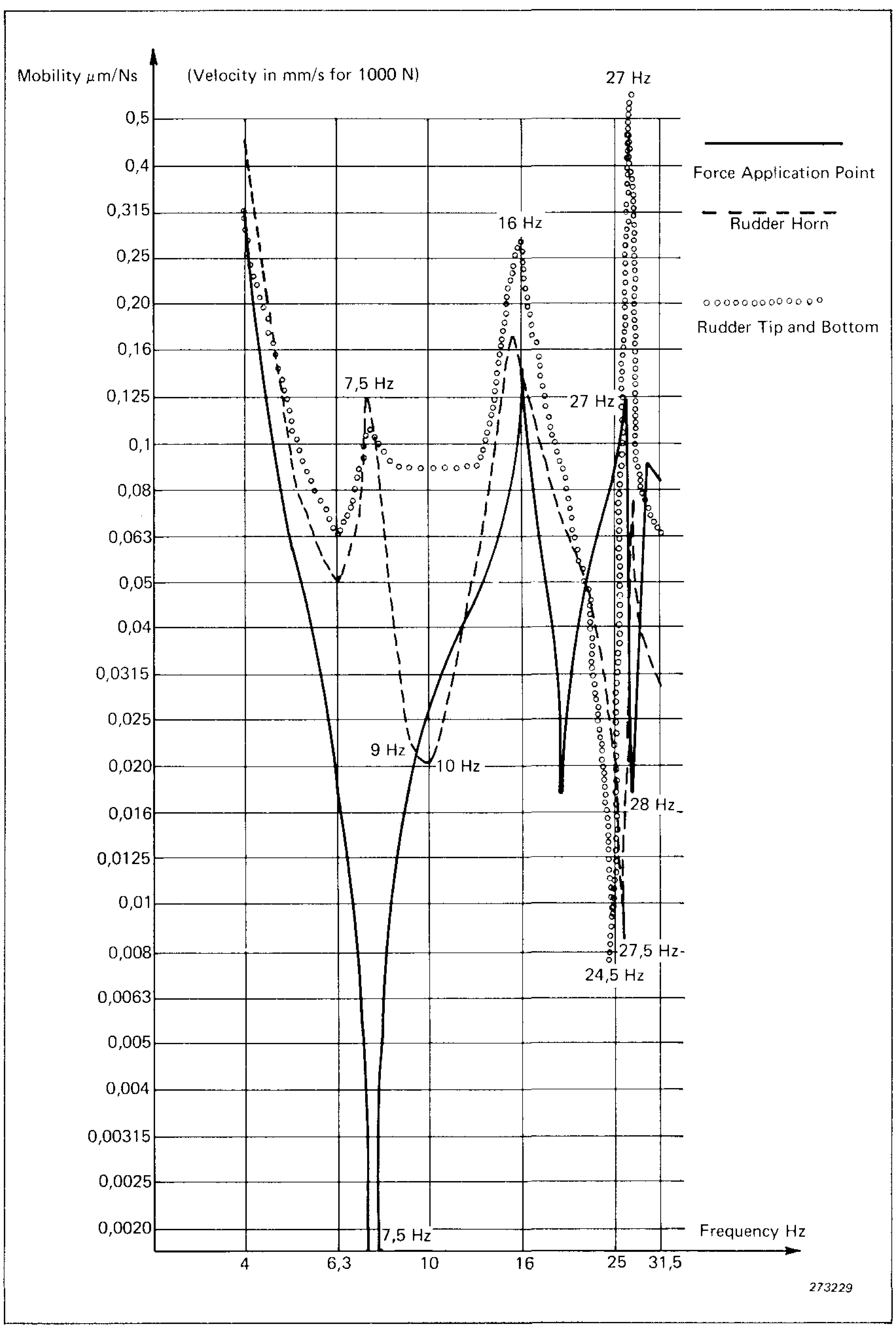


Fig.16. The original recordings transformed to a mobility plot of point and transfer mobilities

200 kg with 7,25 m/s² and the nearest corresponding line (with slope —1) is found in Fig.19. This line is followed to the left until it crosses the line for 19 mm displacement (slope + 1) at app. 3 Hz. If vibration must be carried out at lower frequencies, either the force must be reduced, or extra mass must be added to the shaker body to reduce its acceleration and, thereby, provide full force at low frequencies without crossing the max. displacement line (19 mm) for the Mode Study Head.

References

1. J. P. Salter, "Steady State Vibration". Kenneth Mason, Hampshire, England, 1969
2. USAS S2.6-1963, "Specifying the Mechanical Impedance of Structures".
3. H. P. Olesen and R. B. Randall, "A Guide to Mechanical Impedance and Structural Response Techniques". Brüel & Kjær Application Note No. 17—179.
4. K. Zaveri and H. P. Olesen, "Measurement of Elastic Modulus and Loss Factor of Asphalt". Technical Review No. 4, 1972.
5. Jens Trampe Broch, "Mechanical Vibration and Shock Measurements". (Revised Edition, May 1972) Brüel & Kjær.

Further Literature

6. Torben Licht, "Measurement of Low Level Vibration in Buildings". Technical Review No. 3, 1972.
7. Willy Brænder, "High Fre- Response of Force Transducers" Technical Review No. 3, 1972.
8. D. J. Ewins, "Some whys and wherefores of impedance testing". See dynamic testing symposium, Jan. 5—6, 1971, London.
9. Am. Society Mech. Engineers, "Colloquim on Mechanical Impedance Methods for Mechanical Vibrations". ASME meeting N.Y. Dec. 1958.
10. N. F. Hunter, Jr. and J. V. Otts, "The measurement of mechanical impedance and its use in vibration testing". U.S. NAV RES LAB, Shock Vib. Bull 42 pt. 1. Jan. 1972, p. 55 — 69.

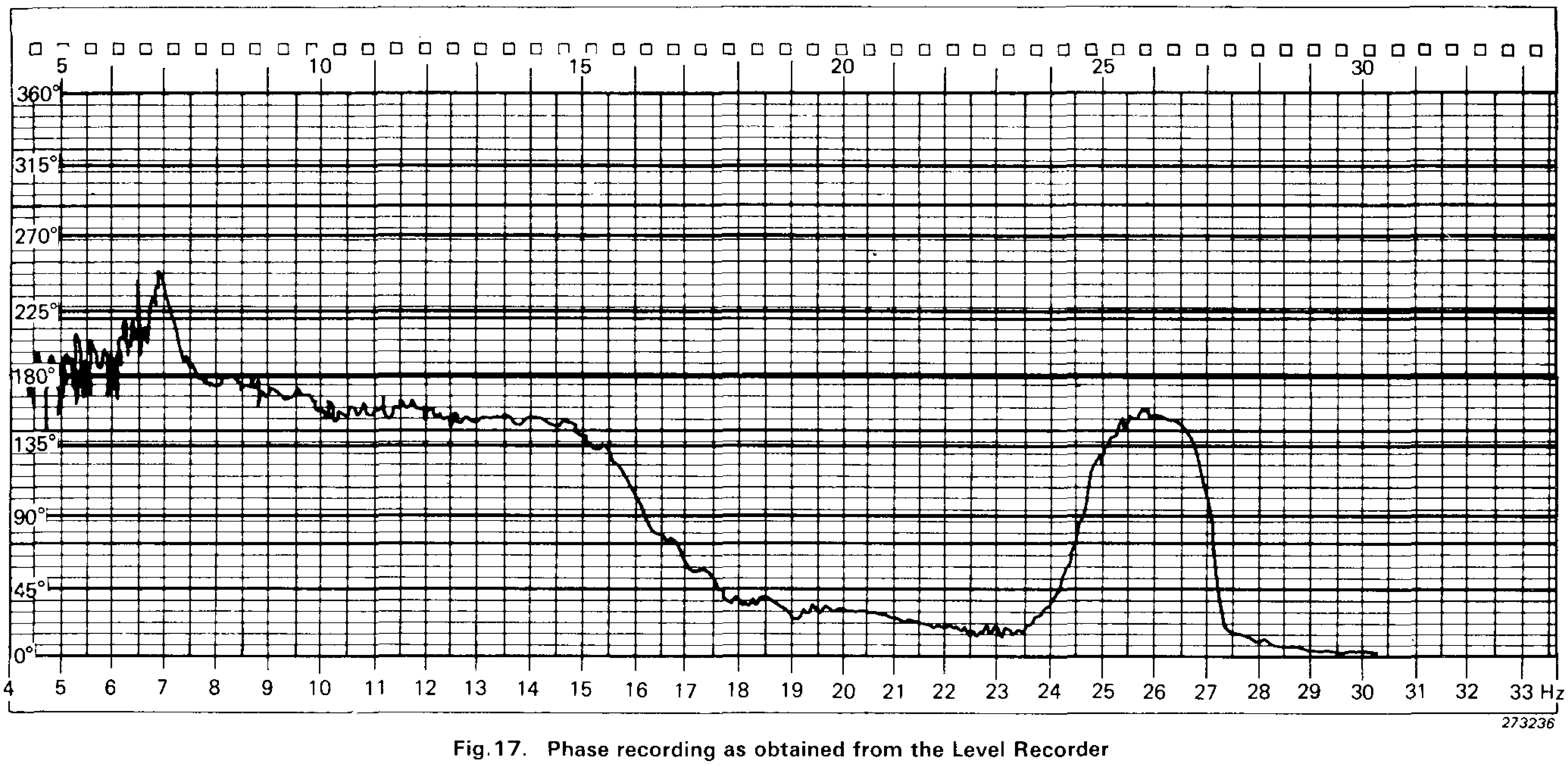


Fig.17. Phase recording as obtained from the Level Recorder

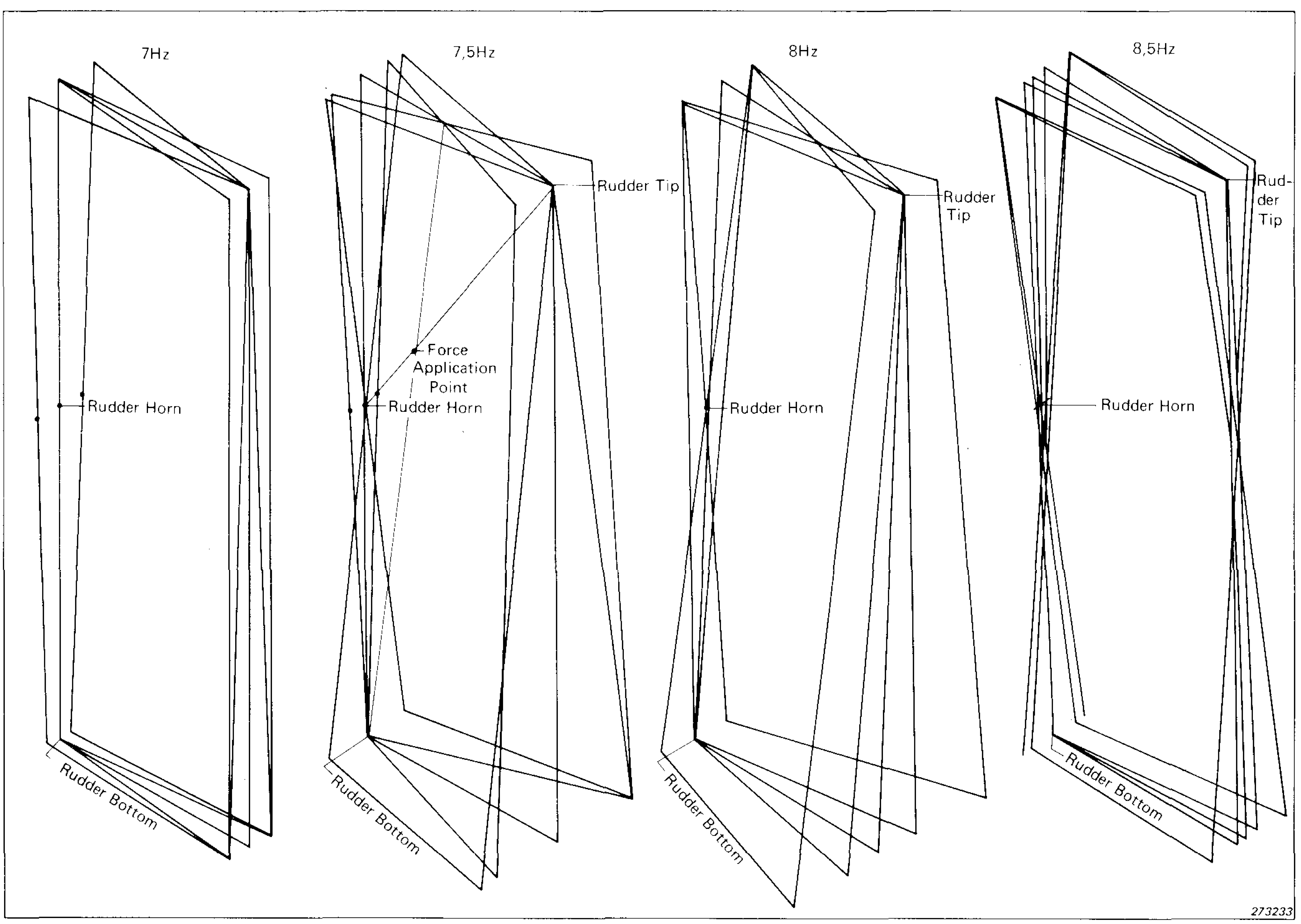


Fig.18. Diagrams of acceleration relationships (or displacement) as the frequency is varyed over the antiresonance at 7,5 Hz

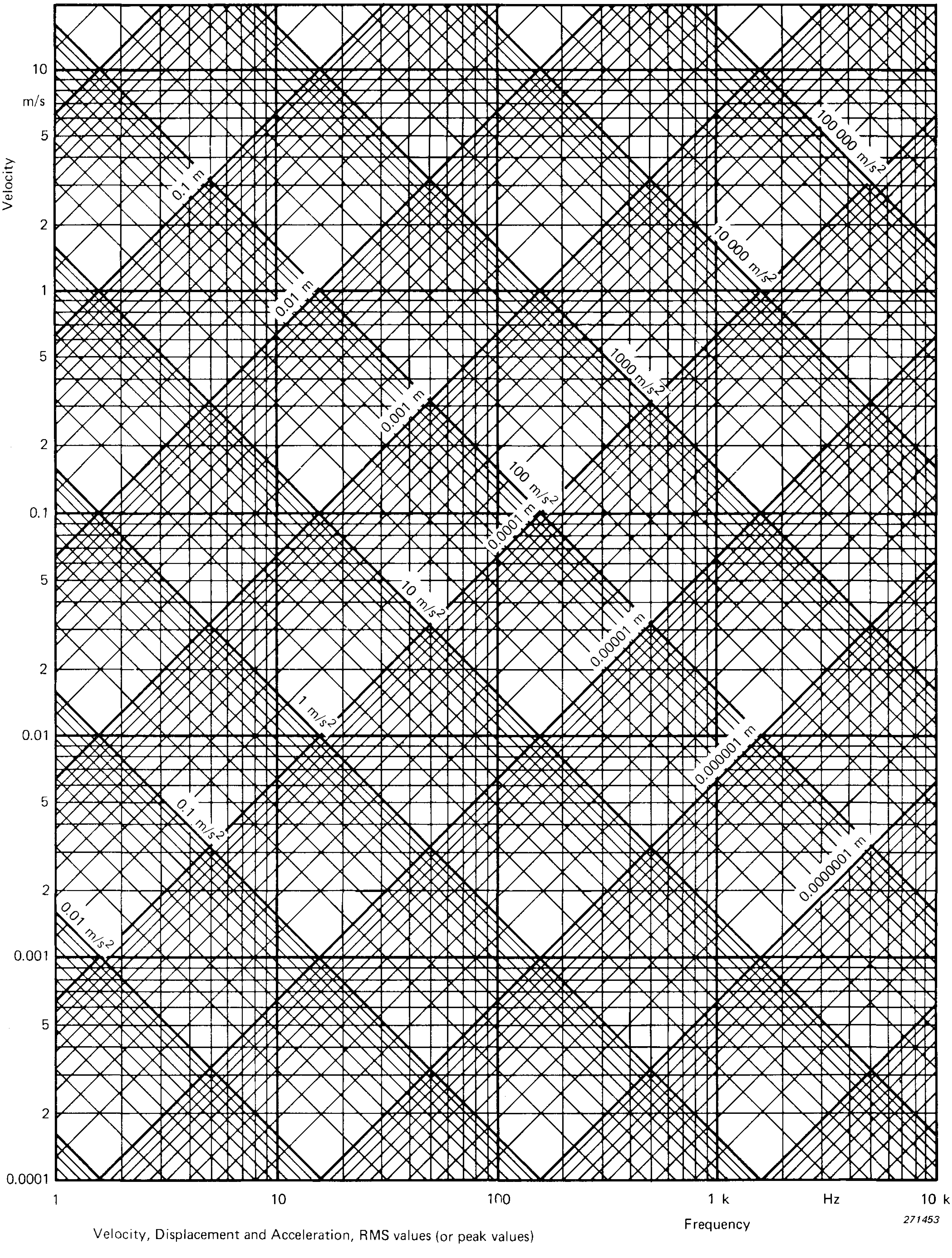


Fig. 19. ***Nomograph of the relations of sinusoidal acceleration, velocity and displacement as function of frequency***

Brüel & Kjær

DK-2850 NÆRUM, DENMARK
Telephone: + 45 2 80 05 00
TELEX: 37316 bruka dk

Printed in Denmark by Nærum Offset